



The Many Functions of Quantitative Modeling

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Abstract

Lee et al. (2019) recently proposed a number of quantitative modeling techniques and practices culminating with a recommendation for the creation of registered model reports. In addition to increasing transparency, trust, and robustness of modeling practices, registered model reports seem likely to increase the visibility and dissemination of quantitative models to researchers from other scientific disciplines (including other behavioral and non-behavioral sciences). In addition to the recommendations proposed by Lee et al. (2019), interdisciplinary communication and collaboration will be improved if researchers include the function of the quantitative model within registered model reports. Here, the function of a quantitative model refers to the contexts (e.g., questions, data types) and goals (e.g., description, prediction, exploration) surrounding the scientist's use of a model. Explicitly specifying the function of a model will allow for more accurate and fair tests of quantitative models and appropriate tests of model generalizability to novel research questions.

Keywords Quantitative analyses · Interdisciplinary collaboration · Verbal behavior · Language

I love quantitative modeling of behavior and I'm not a cognitive scientist. I admit to thoroughly enjoying the moment where a curve defined by a simple equation snaps into place on top of the data following parameter estimation (Nevin 2008). It is awesome (in all the original meaning of the word) that the seeming complexity of an organism's experience can be described by quantitative relations between a few independent and dependent variables. My guess is that readers of this journal feel a similar exhilaration when modeling.

Also, my guess is the readers of this journal use quantitative models for many different purposes. Thus, though I support the key ideas put forth by Lee et al. (2019), I would add an additional recommendation that registered modeling reports include the function of the researcher's modeling behavior. Creating registered reports of quantitative models developed by cognitive scientists may increase the visibility of the models as well as the frequency that non-cognitive scientists (such as myself) contact the models. Explicitly stating the function of a quantitative model may allow for the more accurate evaluation of quantitative models, easier tests of model

generalizability, and more effective interdisciplinary communication and collaboration. I now turn to what is meant by a functional description of modeling.

A Functional Description of Modeling

All models specify relations among dependent variables and one or more independent variables. These relations are often specified using words (verbal models) or mathematics (quantitative models). Verbal models have the benefit of interpretability to the mathematically uninclined or researchers from outside the discipline where a model is derived. Quantitative models have the benefits of increased precision in describing phenomena, more specific predictions, easier falsifiability of theory, and unification of diverse phenomena (Dallery and Soto 2013).

Nevertheless, both words and mathematics are instances of human behavior (Marr 2015), and all behavior has some function (Baum 2018)—some reason why the behavior occurs. Thus, to really understand a quantitative model, researchers also need to know the contexts in which it should (and should not) be used, and the scope and type of answers that plausibly result from using the model. The context, modeling behavior, and modeling outcomes form a single, functional unit (Fig. 1). Scientists cannot separate the use of a model to describe or

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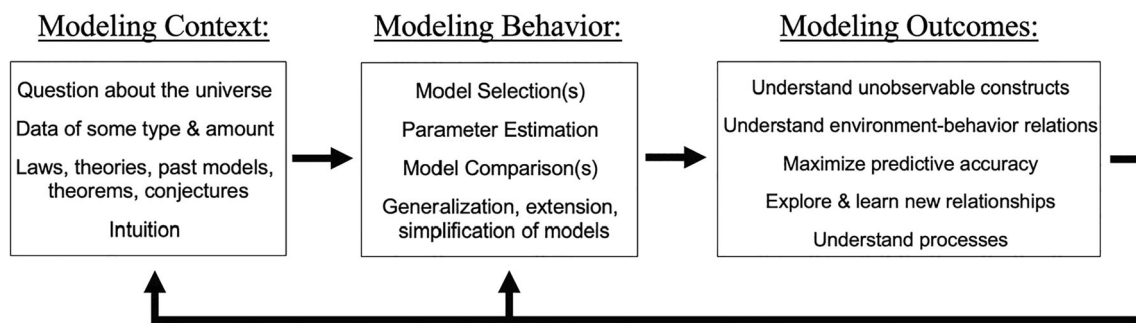


Fig. 1 Visual model of a functional description of quantitative modeling. Many aspects of quantitative modeling are influenced by the context and outcomes that surround quantitative modeling. Modeling outcomes then influence future instances of quantitative modeling. Registered model

interpret data, or the act of deriving a new model, from the context and goals surrounding the behavior of modeling.

Some Factors That May Influence the Function of Modeling

The function of quantitative modeling can vary depending on the question. Lee et al. (2019) note they tend to focus on psychological models where parameters can be interpreted as unobservable constructs. This differs from measurement models aimed at developing probability estimates of psychological processes and tests of model assumptions (e.g., Erdfelder et al. 2009). Both of these functions differ from quantitative modeling in my research focused on descriptions of environment-behavior relations, interpretation of controlling variables at the environment-behavior level (Moore 2015), and how parameters within these models are interpreted (MacCorquodale and Meehl 1948). All three functions described thus far differ from a fourth function of quantitative models in maximizing predictive accuracy (e.g., some machine learning and statistical models; Durstewitz et al. 2019; Walsh et al. 2018; Zhou et al. 2018). Finally, all of these differ from a fifth function identified by Lee et al. (2019) in exploratory modeling and attempts to learn some new relationship between variables in collected data.

The function of quantitative modeling can also vary depending on the object and level of study. For example, quantitative models may focus on describing behavior at the group level or at the individual level (Regenwetter and Robinson 2017). This distinction is important. For example, cumulative prospect theory is used to describe the choice between two risky alternatives, and estimated group parameters often indicate that a majority of participants overweight small probabilities for gains (Tversky and Kahneman 1992). But it would be absurd to say quantitative models that describe a proportion of participants also describe individual responding. By definition, a proportion of a sample means that some participants are not engaged in the same processes and an account of

reports are likely to have greater utility if they include the context (e.g., questions, data types) and intended outcomes (e.g., description, prediction, exploration) for which a model was designed

behavior at the individual level cannot be contained in group-level descriptions (Regenwetter and Robinson 2017). Similarly, delay discounting is a common finding where increasing the delay to some event (e.g., receiving money) reduces the present value of that event (e.g., Odum 2011). A quantitative model of the relations between environmental variables of delay, outcome amount, and individual behavior has been robustly supported by research (e.g., McKerchar and Renda 2012; Vanderveldt et al. 2016). But these descriptions of environment-behavior relations at the individual level provide no indication of the neurobiological mechanisms of discounting, nor provide a rationale for why delay discounting differs between groups of people (e.g., between substance users and non-users; Amlung et al. 2017).

The specific context and research goals in each scenario above will likely influence the modeling behavior of the scientist. The scientists have found themselves with a particular set of data, in a particular context, and seeking answers to a particular question. Explicitly identifying the context and outcomes sought (or plausible) is important because the particular function of modeling may change: what constitutes “good modeling practices,” appropriate evaluation of a model, appropriate attempts to generalize a model, how free-parameters are interpreted, the conditions in which a model should be compared to data, and which models can be legitimately compared.

Some Benefits of Including Modeling Function in Registered Modeling Reports

Quantitative modeling can be difficult and researchers may read broadly outside their home discipline for inspiration, helpful tools, and novel approaches. For example, Newton’s laws of motion were used as inspiration to study and model changes in operant behavior (e.g., Nevin et al. 1981, 1983; Hubbard 2017). A model that describes how individual members of a group of conspecifics distribute across patches with different resource densities (e.g., Fretwell and Lucas 1970;

Kraft et al. 2002) bears a striking similarity to a model that describes how individual responses in an organism's behavioral repertoire distribute across response options with different reinforcer densities (e.g., Herrnstein 1961, 1970). And, models that identify patterns of optimal decision-making can aide our understanding of where and how humans behave sub-optimally (e.g., Chaturvedi et al. 1993; Kolodner 1991; Meyer et al. 2014). In addition to inspiration and translation to novel domains, some researchers are bridging modeling domains in hybrid fashion such as using machine learning to guide cognitive modeling (e.g., Agrawal et al. 2019) or learning cognitive models using machine learning (e.g., Chaplot et al. 2018).

Creating registered modeling reports will likely increase the accessibility and dissemination of quantitative models from a variety of disciplines. The languages of mathematics and computation (Marr 2015) provide a common language for researchers with varied backgrounds and experiences to communicate about the world and take advantage of the work conducted in other scientific domains. Additionally, the human ability of metaphors (e.g., Skinner 1957; Stewart and Barnes-Holmes 2001) may allow researchers to draw connections and analogies between a model from one discipline and a phenomenon from their own discipline. As demonstrated above, this can lead to tremendous benefits for a scientific domain when done well.

But shared language and metaphor can be a double-edged sword. On one hand, shared language may allow models to be easily used in novel settings which may aid direct tests of prediction and generalization of models to novel problems. On the other hand, without understanding the function and underlying assumptions of a model, it may be used inappropriately and falsely rejected as ineffective. Researchers from varied disciplines are more likely to fairly test and generalize a model if a registered model report includes information on the function of the model, the conditions under which it performs well, the conditions under which it performs poorly, and what models serve similar functions and can be used for legitimate comparison. One can envision a future where a variety of behavioral, cognitive, and artificial intelligence models are catalogued in a single location based on function and with links to relevant tests of validation and generalization. Researchers and industry professionals could then peruse and identify the most appropriate model based on the context and goals of their research question, explore related families of models from multiple scientific disciplines, and extend past research in novel directions—all classified by a model function.

Conclusion

I enthusiastically support the key ideas set forth by Lee et al. (2019). I agree that preregistering models, postregistering

models, undertaking detailed evaluations of models, and creating registered model reports will likely improve the transparency, trust, and robustness of quantitative models describing human and non-human behavior. Importantly, registered model reports seem likely to bring researchers from non-cognitive science disciplines into greater contact with the quantitative and computational models derived by cognitive scientists. These researchers may be seeking inspiration, tools, or approaches to help solve problems in their own disciplines. Thus, to the recommendations set forth by Lee et al. (2019), I add a suggestion to include explicit statements of the function of quantitative models in registered model reports. Including the function of the quantitative model relative to the type of question and object of study will aide in effective interdisciplinary communication and collaboration, and also will allow for accurate and fair tests of model prediction and generalizability to novel research questions beyond the model's original function.

Compliance with Ethical Standards

Conflict of Interest The author declares no conflict of interest.

References

- Agrawal, M., Peterson, J.C., & Griffiths, T.L. (2019). Using machine learning to guide cognitive modeling: a case study in moral reasoning. <https://arxiv.org/pdf/1902.06744.pdf>
- Amlung, M., Vedelago, L., Acker, J., Balodis, I., & MacKillop, J. (2017). Steep delay discounting and addictive behavior: a meta-analysis of continuous associations. *Addiction*, *112*, 51–62. <https://doi.org/10.1111/add.13535>.
- Baum, W. M. (2018). Three laws of behavior: allocation, induction, and covariance. *Behavior Analysis: Research and Practice*, *18*, 239–251. <https://doi.org/10.1037/bar0000104>.
- Chaplot, D., MacLellan, C., Salakhutdinov, R., & Koedinger, K. (2018). Learning cognitive models using neural networks. <https://arxiv.org/pdf/1806.08065.pdf>
- Chaturvedi, A. R., Hutchinson, G. K., & Nazareth, D. L. (1993). Supporting complex real-time decision making through machine learning. *Decision Support Systems*, *10*, 213–233. [https://doi.org/10.1016/0167-9236\(93\)90039-6](https://doi.org/10.1016/0167-9236(93)90039-6).
- Dallery, J., & Soto, P. L. (2013). Quantitative description of environment-behavior relations. In G. J. Madden (Ed.), *APA handbook of behavior analysis: Vol. 1. Methods and principles* (pp. 219–249). Washington DC: American Psychological Association.
- Durstewitz, D., Koppe, G., & Meyer-Lindenberg, A. (2019). Deep neural networks in psychiatry. *Molecular Psychiatry*, (ePub ahead of print). <https://doi.org/10.1038/s41380-019-0365-9>.
- Erdfelder, E., Auer, T. S., Hilbig, B. E., Abfal, A., Moshagen, M., & Nadarevic, L. (2009). Multinomial processing tree models: a review of the literature. *Journal of Psychology*, *217*, 108–124. <https://doi.org/10.1027/0044-3409.217.3.108>.
- Fretwell, S. D., & Lucas, H. L. (1970). On territorial behavior and other factors influencing habitat distribution in birds. I. Theoretical development. *Acta Biotheoretica*, *19*, 16–36.
- Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of the Experimental*

- Analysis of Behavior*, 4, 267–272. <https://doi.org/10.1901/jeab.1961.4-267>.
- Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior*, 13, 243–266.
- Hubbard, T. L. (2017). Toward a general theory of momentum-like effects. *Behavioural Processes*, 141, 50–66. <https://doi.org/10.1016/j.beproc.2017.02.019>.
- Kolodner, J. L. (1991). Improving human decision making through case-based decision aiding. *AI Magazine*, 12, 52–68. <https://doi.org/10.1609/aimag.v12i2.895>.
- Kraft, J. R., Baum, W. M., & Burge, M. J. (2002). Group choice and individual choices: Modeling human social behavior with the Ideal Free Distribution. *Behavioural Processes*, 57, 227–240. [https://doi.org/10.1016/S0376-6357\(02\)00016-5](https://doi.org/10.1016/S0376-6357(02)00016-5).
- Lee, MD, Criss, AH, Devezer, B, Donkin, C, Etz, A, Leite FP, Matzke D, Rouder JN, Trueblood JS, White CN, & Vandekerckhove, J (2019). Robust modeling in cognitive science. *Computational Brain & Behavior*, ePub ahead of print. <https://doi.org/10.1007/s42113-019-00029-y>.
- MacCorquodale, K., & Meehl, P. E. (1948). On a distinction between hypothetical constructs and intervening variables. *Psychological Review*, 55, 95–107. <https://doi.org/10.1037/h0056029>.
- Marr, M. J. (2015). Reprint of “Mathematics as verbal behavior”. *Behavioural Processes*, 114, 34–40. <https://doi.org/10.1016/j.beproc.2015.03.008>.
- McKerchar, T. L., & Renda, C. R. (2012). Delay and probability discounting in humans: an overview. *The Psychological Record*, 62, 817–834. <https://doi.org/10.1007/BF03395837>.
- Meyer, G., Adomavicius, G., Johnson, P. E., Elidrisi, M., Rush, W. A., Sperl-Hillen, J. M., & O’Connor, P. J. (2014). A machine learning approach to improving dynamic decision making. *Information Systems Research*, 25, 239–263. <https://doi.org/10.1287/isre.2014.0513>.
- Moore, J. (2015). Pragmatism, mathematical models, and the scientific ideal of prediction and control. *Behavioural Processes*, 114, 2–13. <https://doi.org/10.1016/j.beproc.2015.01.007>.
- Nevin, J. A. (2008). Control, prediction, order, and the joys of research. *Journal of the Experimental Analysis of Behavior*, 89, 119–123. <https://doi.org/10.1901/jeab.2008.89-119>.
- Nevin, J. A., Mandell, C., & Yarinsky, P. (1981). Response rate and resistance to change in chained schedules. *Journal of Experimental Psychology: Animal Behavior Processes*, 7, 278–294.
- Nevin, J. A., Mandell, C., & Yarinsky, P. (1983). The analysis of behavioral momentum. *Journal of the Experimental Analysis of Behavior*, 39, 49–59.
- Odum, A. L. (2011). Delay discounting: I’m a *k*, you’re a *k*. *Journal of the Experimental Analysis of Behavior*, 96, 427–439. <https://doi.org/10.1901/jeab.2011.96-423>.
- Regenwetter, M., & Robinson, M. M. (2017). The construct-behavior gap in behavioral decision research: a challenge beyond replicability. *Psychological Review*, 124, 533–550. <https://doi.org/10.1037/rev0000067>.
- Skinner, B. F. (1957). *Verbal behavior*. New York, NY: Appleton-Century-Crofts.
- Stewart, I., & Barnes-Holmes, D. (2001). Understanding metaphor: a relational frame theory perspective. *The Behavior Analyst*, 24, 191–199.
- Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5, 297–323.
- Vanderveldt, A., Oliveira, L., & Green, L. (2016). Delay discounting: pigeon, rate, human – does it matter? *Journal of Experimental Psychology: Animal Learning and Cognition*, 42, 141–162. <https://doi.org/10.1037/xan0000097>.
- Walsh, C. G., Ribeiro, J. D., & Franklin, J. C. (2018). Predicting suicide attempts in adolescents with longitudinal clinical data and machine learning. *Journal of Child Psychology and Psychiatry*, 59, 1261–1270. <https://doi.org/10.1111/jcpp.12916>.
- Zhou, M., Yoshimi Fukuoka, Y., Mintz, Y., Goldberg, K., Kaminsky, P., Flowers, E., & Aswani, A. (2018). Evaluating machine learning-based automated personalized daily step goals delivered through a mobile phone app: randomized controlled trial. *JMIR mHealth and uHealth*, 6, e28. <https://doi.org/10.2196/mhealth.9117>.

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